Differential Equations

LAB 1

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**Question 1**

Code

f = @(t,y) t - (y^2)

quiver244(f, 0, 5,-3,10,1,'r')

Figure 1



Figure 2

****

Comment

What does it look like the solution will do if we start with y(0) = 3?

The Solution will decrease and then deviate upwards as t increases and y decreases

What about for y(0) = −1?

The Solution will start deviating downwards as y decreases and t increases

Use samplePlots244.m to verify your results.

y(0) = 3



y(0) = -1



**Question 2**

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y(0)= -6, y(0)= -5, y(0)= -4, y(0)= -3, y(0)= -2, y(0)= -1, y(0)= 0, y(0)= 1, y(0)= 2, y(0)= 3, y(0)= 4, y(0)= 5



Comment

For y(0) at -6, the solution remains the same as t gets large

For y(0) between -5 and -3, the solution goes to -2 as t gets large

For y(0) at -2, the solution remains the same as t gets large

For y(0) at -1, the solution goes to 0 as t gets large

For y(0) at 0, the solution remains the same as t gets large

For y(0) between 4 and 1, the solution decreases to 0 as t gets large

For y(0) at 5, the solution remains the same as t gets large

**Question 3**

Code

function [t,y] = eulerMethod(f, dt, Tf, t0, y0)

t = t0:dt:Tf;

y = zeros(size(t));

y(1)=y0;

n = length(y);

for i = 1:(n-1)

E = f(t(i),y(i));

y(i+1) = y(i) + dt\*E;

end

**Question 4**

Delta t = 1

[t,y] = eulerMethod(f,1,10,0,1)

Elapsed time is 0.000100 seconds.

t =

0 1 2 3 4 5 6 7 8 9 10

y =

1.0e+04 \*

Columns 1 through 8

0.0001 0.0002 0.0004 0.0009 0.0026 0.0077 0.0220 0.0618

Columns 9 through 11

0.1707 0.4680 1.2774

Delta t = 0.1

[t,y] = eulerMethod(f,0.1,10,0,1)

Elapsed time is 0.003220 seconds.

t =

Columns 1 through 8

0 0.1000 0.2000 0.3000 0.4000 0.5000 0.6000 0.7000

Columns 9 through 16

0.8000 0.9000 1.0000 1.1000 1.2000 1.3000 1.4000 1.5000

Columns 17 through 24

1.6000 1.7000 1.8000 1.9000 2.0000 2.1000 2.2000 2.3000

Columns 25 through 32

2.4000 2.5000 2.6000 2.7000 2.8000 2.9000 3.0000 3.1000

Columns 33 through 40

3.2000 3.3000 3.4000 3.5000 3.6000 3.7000 3.8000 3.9000

Columns 41 through 48

4.0000 4.1000 4.2000 4.3000 4.4000 4.5000 4.6000 4.7000

Columns 49 through 56

4.8000 4.9000 5.0000 5.1000 5.2000 5.3000 5.4000 5.5000

Columns 57 through 64

5.6000 5.7000 5.8000 5.9000 6.0000 6.1000 6.2000 6.3000

Columns 65 through 72

6.4000 6.5000 6.6000 6.7000 6.8000 6.9000 7.0000 7.1000

Columns 73 through 80

7.2000 7.3000 7.4000 7.5000 7.6000 7.7000 7.8000 7.9000

Columns 81 through 88

8.0000 8.1000 8.2000 8.3000 8.4000 8.5000 8.6000 8.7000

Columns 89 through 96

8.8000 8.9000 9.0000 9.1000 9.2000 9.3000 9.4000 9.5000

Columns 97 through 101

9.6000 9.7000 9.8000 9.9000 10.0000

y =

1.0e+04 \*

Columns 1 through 8

0.0001 0.0001 0.0001 0.0001 0.0001 0.0002 0.0002 0.0002

Columns 9 through 16

0.0002 0.0002 0.0002 0.0002 0.0003 0.0003 0.0003 0.0003

Columns 17 through 24

0.0004 0.0004 0.0004 0.0005 0.0005 0.0006 0.0006 0.0007

Columns 25 through 32

0.0008 0.0009 0.0010 0.0011 0.0012 0.0013 0.0015 0.0017

Columns 33 through 40

0.0018 0.0021 0.0023 0.0026 0.0029 0.0032 0.0036 0.0040

Columns 41 through 48

0.0044 0.0049 0.0055 0.0061 0.0068 0.0076 0.0084 0.0094

Columns 49 through 56

0.0104 0.0116 0.0129 0.0143 0.0159 0.0177 0.0196 0.0218

Columns 57 through 64

0.0242 0.0268 0.0298 0.0330 0.0366 0.0406 0.0450 0.0498

Columns 65 through 72

0.0552 0.0612 0.0678 0.0750 0.0831 0.0920 0.1019 0.1128

Columns 73 through 80

0.1248 0.1381 0.1529 0.1691 0.1872 0.2071 0.2291 0.2534

Columns 81 through 88

0.2803 0.3100 0.3429 0.3792 0.4194 0.4637 0.5128 0.5670

Columns 89 through 96

0.6270 0.6932 0.7665 0.8474 0.9369 1.0358 1.1450 1.2658

Columns 97 through 101

1.3993 1.5469 1.7100 1.8902 2.0894

Delta t = 0.0001

>> eulerMethod(f,0.0001,10,0,1);

Elapsed time is 0.024642 seconds.

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Description automatically generated

Comment

The actual solution at t=10 gives a value of 21,976.46…..

Using the three methods, the last method had the highest accuracy which is a value of 21,975 which is close to the actual value. Method two had better accuracy than method one. Method two value was 20,894 which is closer than 12,774 in method 1.

Method 3 had the slowest running time 0.024642 seconds as it went through more steps of iteration. Method 2 was faster than Method 3 but slower than Method 1 it was 0.003220 seconds. Method 1 0.000100seconds was the fastest as it had least steps to go through.

**Question 5**

****Quiver Plot



Actual Solution Plot

Euler’s Method Plot



**Question 6**

ode23

t =

0

0.0800

0.3740

0.6730

0.9649

1.2495

1.5284

1.8035

2.0766

2.3494

2.6230

2.8983

3.1758

3.4560

3.7389

4.0244

4.3125

4.6030

4.8956

5.1900

5.4861

5.7837

6.0824

6.3822

6.6828

6.9841

7.2860

7.5884

7.8912

8.1943

8.4977

8.8012

9.1050

9.4088

9.7128

10.0000

y =

1.0e+04 \*

0.0001

0.0001

0.0001

0.0002

0.0002

0.0003

0.0003

0.0004

0.0006

0.0008

0.0010

0.0014

0.0019

0.0026

0.0035

0.0048

0.0065

0.0089

0.0122

0.0166

0.0226

0.0308

0.0420

0.0571

0.0776

0.1055

0.1433

0.1946

0.2642

0.3586

0.4867

0.6603

0.8957

1.2150

1.6479

2.1974

Elapsed time is 0.014929 seconds.

ode45

t =

0

0.0502

0.1005

0.1507

0.2010

0.4510

0.7010

0.9510

1.2010

1.4510

1.7010

1.9510

2.2010

2.4510

2.7010

2.9510

3.2010

3.4510

3.7010

3.9510

4.2010

4.4510

4.7010

4.9510

5.2010

5.4510

5.7010

5.9510

6.2010

6.4510

6.7010

6.9510

7.2010

7.4510

7.7010

7.9510

8.2010

8.4510

8.7010

8.9510

9.2010

9.4007

9.6005

9.8002

10.0000

y =

1.0e+04 \*

0.0001

0.0001

0.0001

0.0001

0.0001

0.0001

0.0002

0.0002

0.0003

0.0003

0.0004

0.0005

0.0007

0.0009

0.0011

0.0015

0.0019

0.0026

0.0034

0.0044

0.0058

0.0076

0.0099

0.0129

0.0168

0.0218

0.0283

0.0366

0.0474

0.0613

0.0791

0.1020

0.1315

0.1694

0.2181

0.2807

0.3611

0.4644

0.5971

0.7675

0.9864

1.2053

1.4726

1.7990

2.1976

Elapsed time is 0.157585 seconds.

Comment

ode 45 took more time than ode23 however ode45 gave a more accurate solution than ode23 therefore it took more steps to get a high accurate answer. ode23 time-0.014929, solution-21,974. ode45 time-0.157585, solution-21,976.

Compared to Euler’s method the accuracy for both ode23 and ode45 was high compared to the first two methods in Euler. The running time to get an accurate result in Euler was 0.024642 (third method). ode23 had the least accuracy and the fastest running time. Euler was more accurate than ode23 but had a slower running time. ode24 was the most accurate but had the slowest running time.